Recognise, Sketch and Produce Graphs of Linear and Quadratic Functions of 1 Variable **Answers**

1. Complete the table of values for the function $y = x - 3$

x	-2	-1	0	1	2	3	4
y	-5	**-4**	**-3**	**-2**	**-1**	**0**	1

2. Complete the table of values for the function $y = 3x + 2$

x	-3	-2	-1	0	1	2	3
y	-7	-4	**-1**	**2**	**5**	**8**	**11**

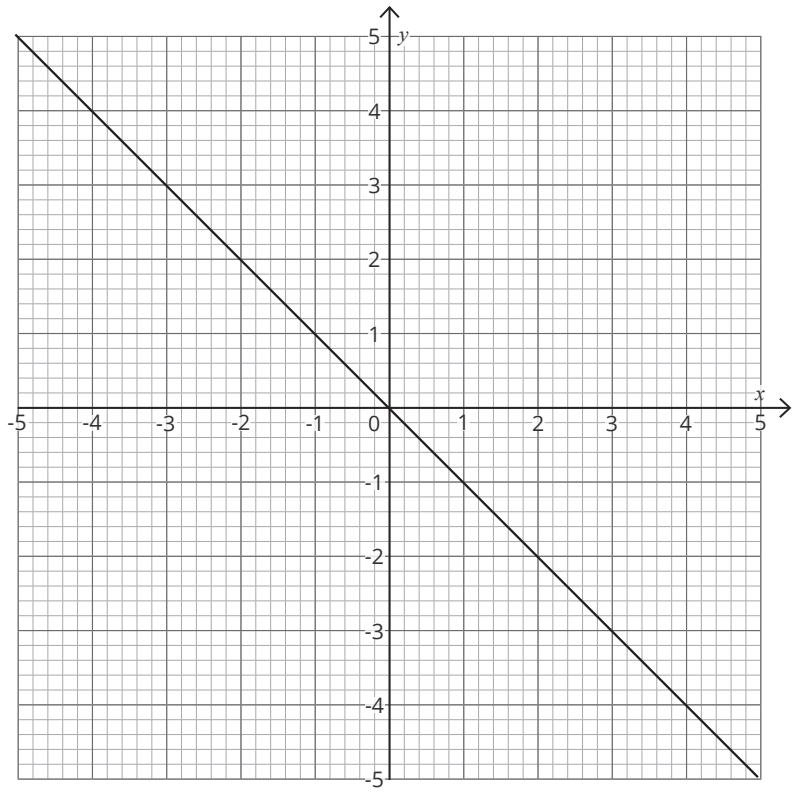
3. Complete the table of values for the function $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	**2**	**0**	**2**	**8**	**18**

4. Complete the table of values for the function $y = 3x^2 - 2$

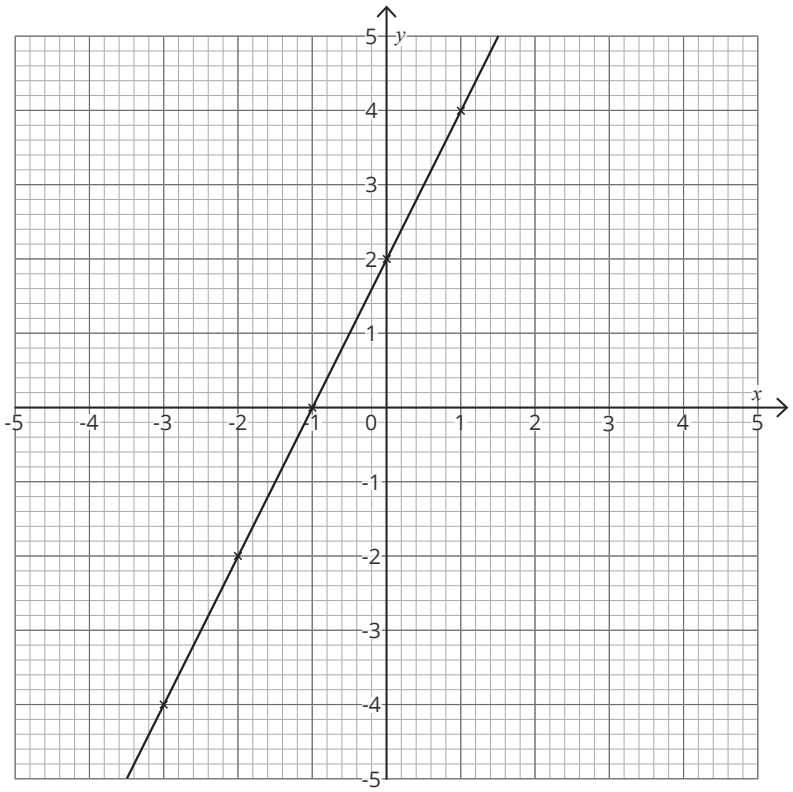
x	-3	-2	-1	0	1	2	3
y	**25**	**10**	**1**	-2	**1**	10	**25**

5. On the grid below, draw the equation of the line $y = -x$.



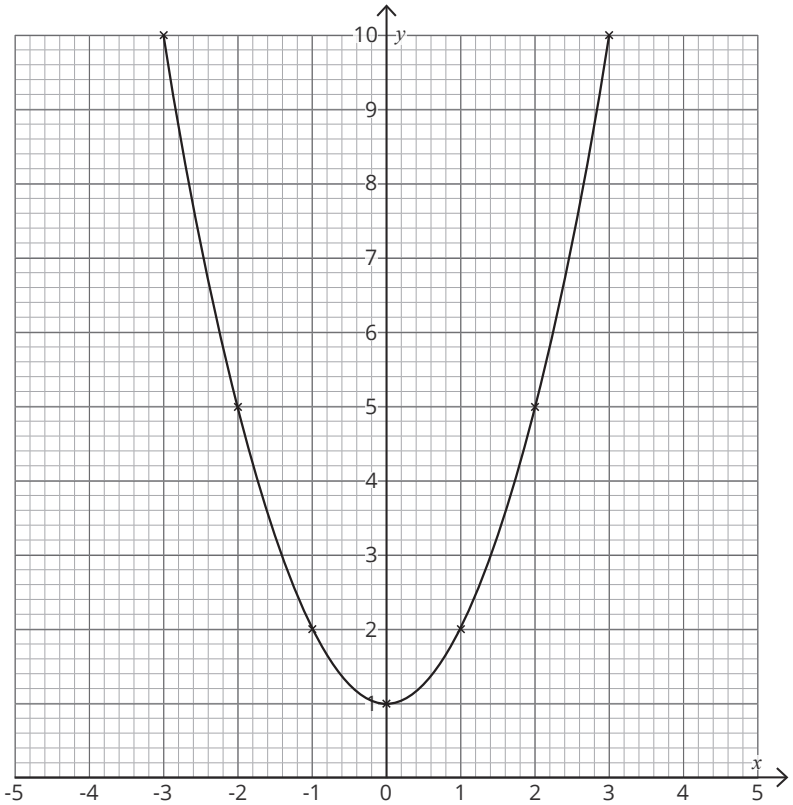
6. Complete the table of values for the function $y = 2x + 2$, then plot the graph on the axes below.

x	-3	-2	-1	0	1	2	3
y	**-4**	**-2**	**0**	**2**	**4**	**6**	**8**

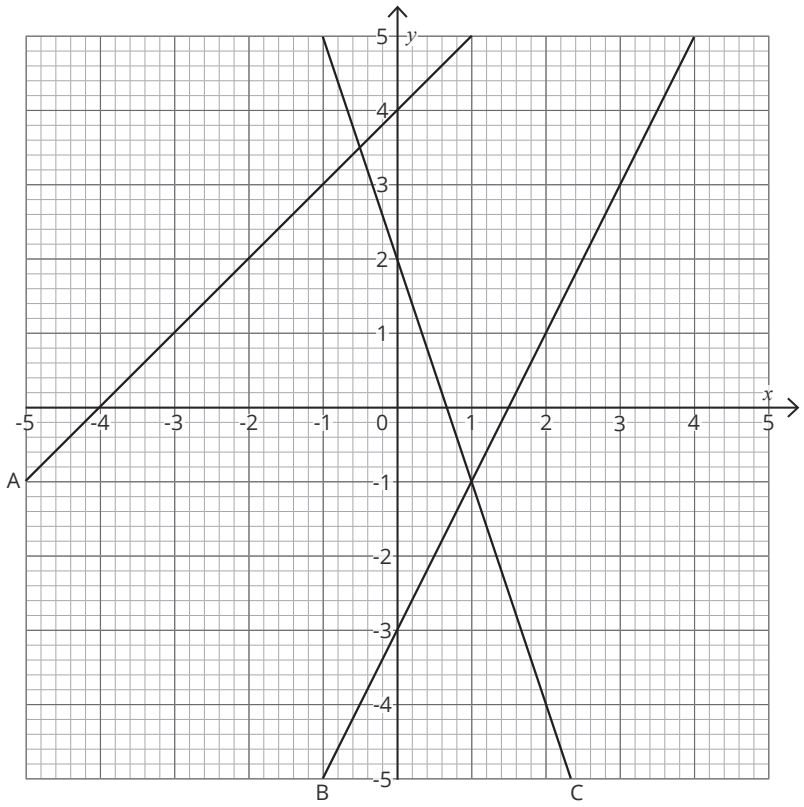


7. Complete the table of values for the function $y = x^2 + 1$, then plot the graph on the axes below.

x	-3	-2	-1	0	1	2	3
y	**10**	**5**	**2**	**1**	**2**	**5**	**10**

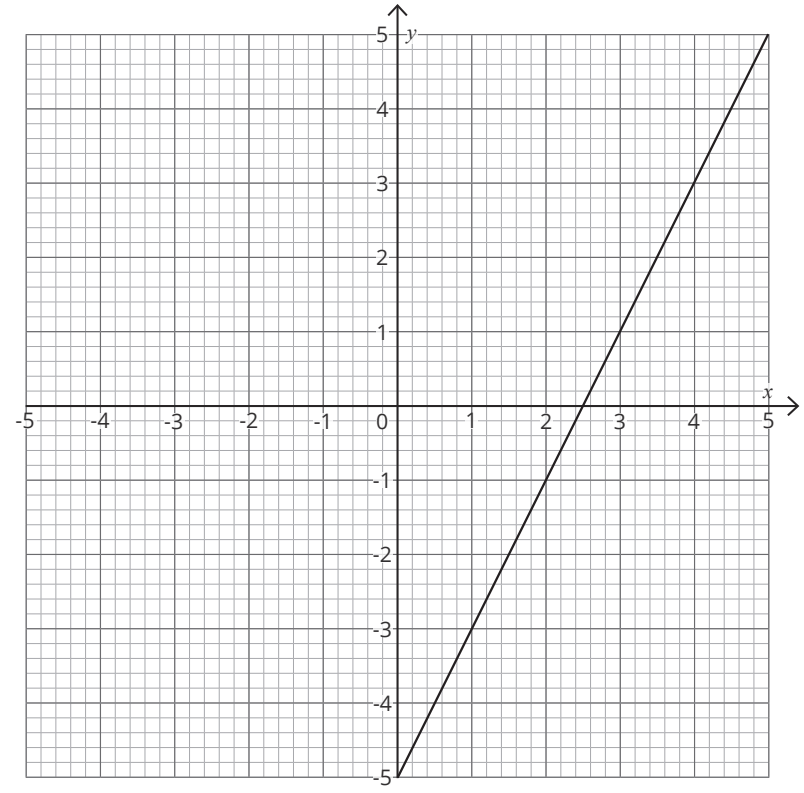


8. Identify the gradient (m) of each line below.



A. $m = 1$ B. $m = 2$ C. $m = -3$

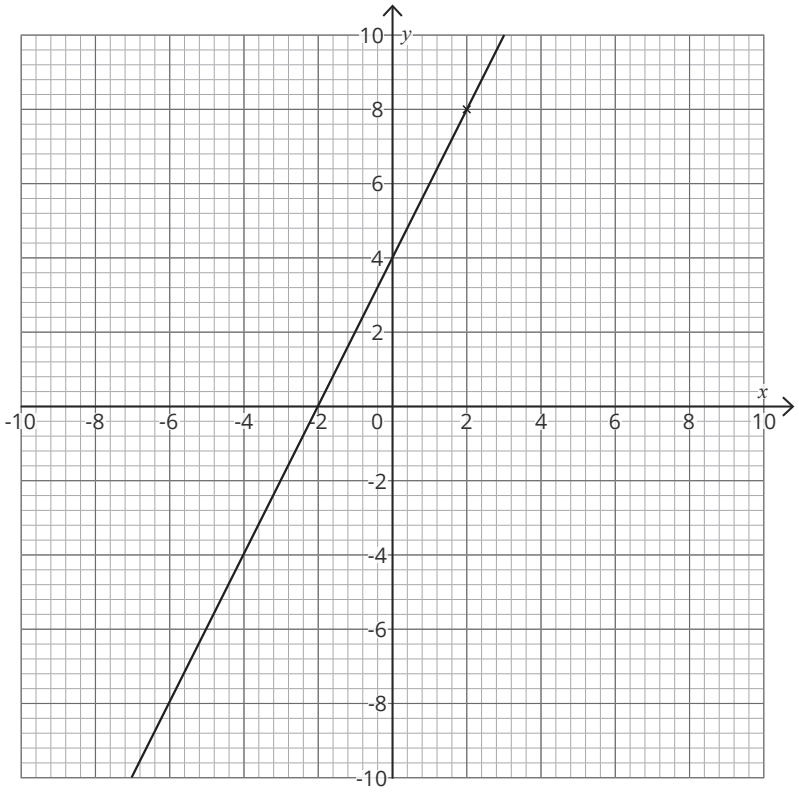
9. Identify the equation of the line.



$y = 2x - 5$

Challenge

Plot a line with a gradient of 2 which crosses the point (2, 8). What is the equation of this line?



$y = 2x + 4$

Recognise, Sketch and Produce Graphs of Linear and Quadratic Functions of 1 Variable

Prior Knowledge:

- Plot coordinates in all 4 quadrants.
- Have knowledge of $y = mx + c$.

A **linear** equation is an equation that contains an algebraic term with a power of 1 but no higher. For example, $y = 3x + 2$ is a linear equation; $y = 3x^2 + 2$ is not. When you draw the graph of a linear equation, you will get a straight line.

A **quadratic** equation is an equation with an algebraic term with a power of 2, but no higher. For example, $y = 5x^2 + 2x + 1$ is a quadratic equation. $y = 5x^3 + 2x + 1$ is not. When you draw the graph of a quadratic equation you will get a parabola – a smooth curve shaped like an n or a u.

Example 1: Plot the lines given by the equations:

a. $y = x$

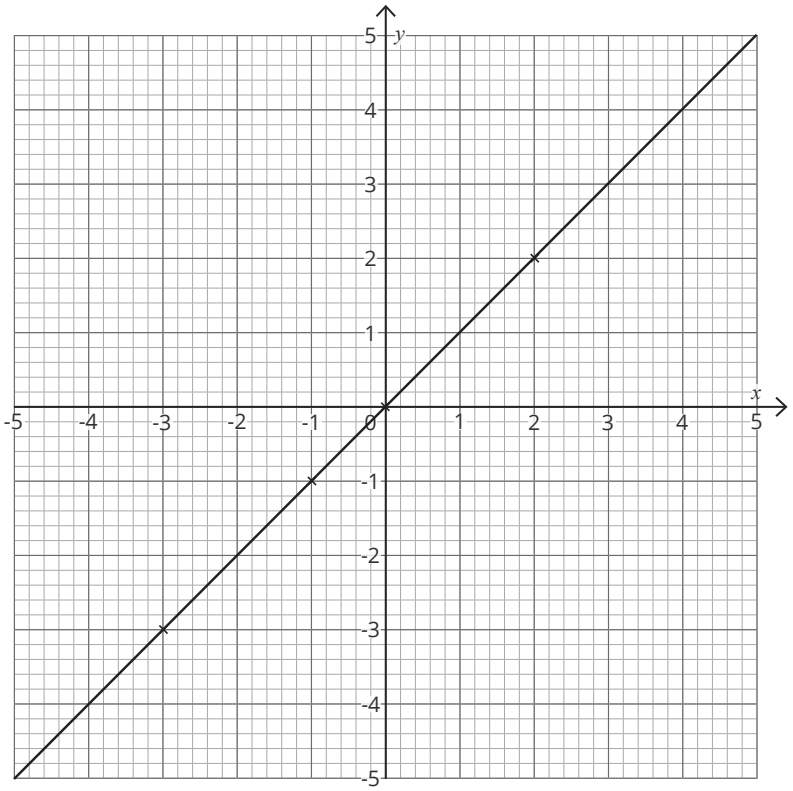
b. $x = 2$

c. $y = -1$

a. $y = x$

In this equation, y is always equal to x. Whatever value of x you choose, y will be the same, for example, coordinates on this line will include (-3, -3), (0, 0), (2, 2), (-1, -1) etc. Pick three of these coordinates, plot them on your axes and use a ruler to join them with a straight line.

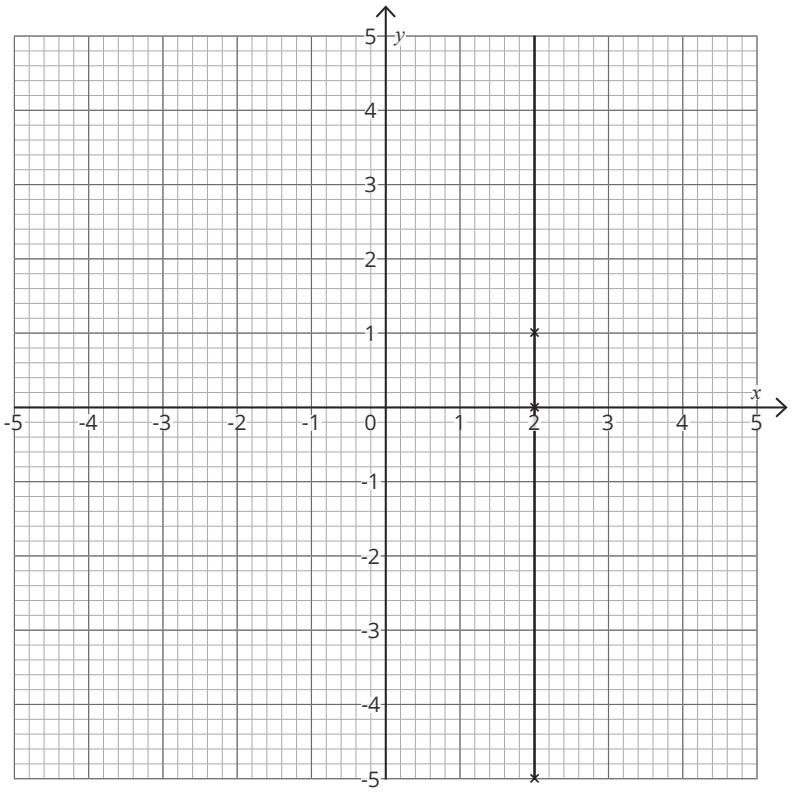
Always draw a line graph with at least three coordinates. This will help you spot if you make a mistake, as the points won't form a straight line. Your line should extend to both edges of the axes – don't just draw a line between the points you have plotted.



b. $x = 2$

In this graph, every x coordinate is 2, regardless of the y coordinate, for example (2, 1), (2, -5), (2, 0) or (2, 652). This will result in a vertical line that crosses the x-axis at 2.

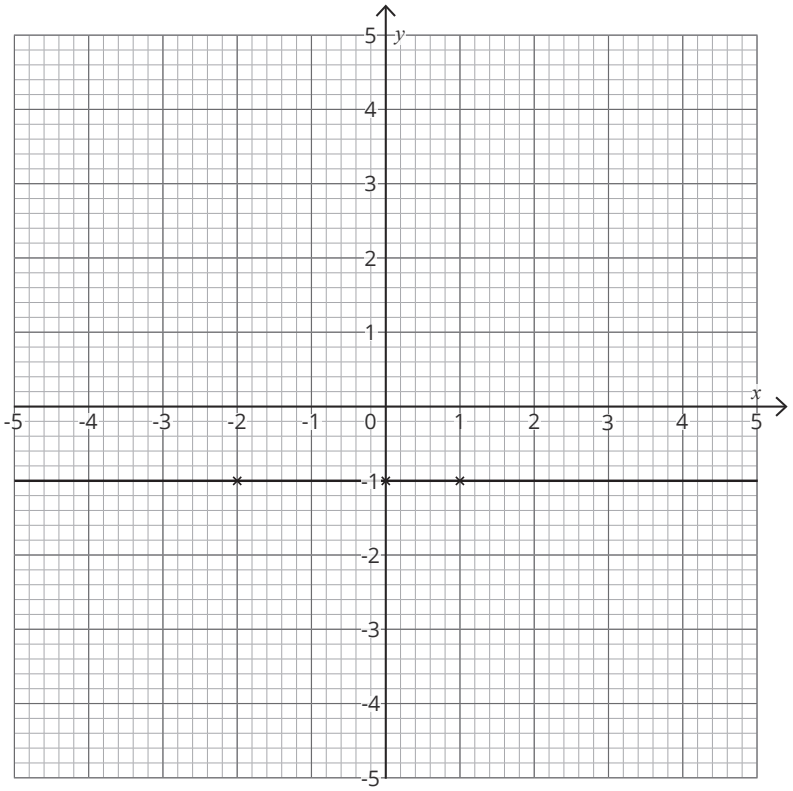
To draw the line, choose 3 coordinates of the form (2, ?) that will fit on your axes, then use your ruler to join them with a straight line. Remember to extend your line to the edge of your axes in both directions.



c. $y = -1$

Similarly, the y-coordinates of every point on this line will be -1, for example, (-2, -1), (0, -1) or (3, -1). This line will cross the y-axis at -1.

As before, plot the 3 coordinates above and use your ruler to join them with a straight line, extending the line to both edges of your axes.



Example 2: Plot the graph given by the equation $y = x^2 - 2$.

When plotting lines or curves from more complex linear or quadratic equations, start by making a table of values. This will give you the coordinates to plot.

Step 1: In many cases, you will be given x-values. If not, start by choosing suitable values for x, based on the values on the axes you have been given. For linear graphs, you need at least 3 x-values. For quadratic graphs, your values should cover the full range of the x-axis.

x	-3	-2	-1	0	1	2	3

Step 2: Using the equation you have been given, substitute each x-value in turn to find the corresponding y-value. For example:

When $x = -3$:

$y = (-3)^2 - 2$

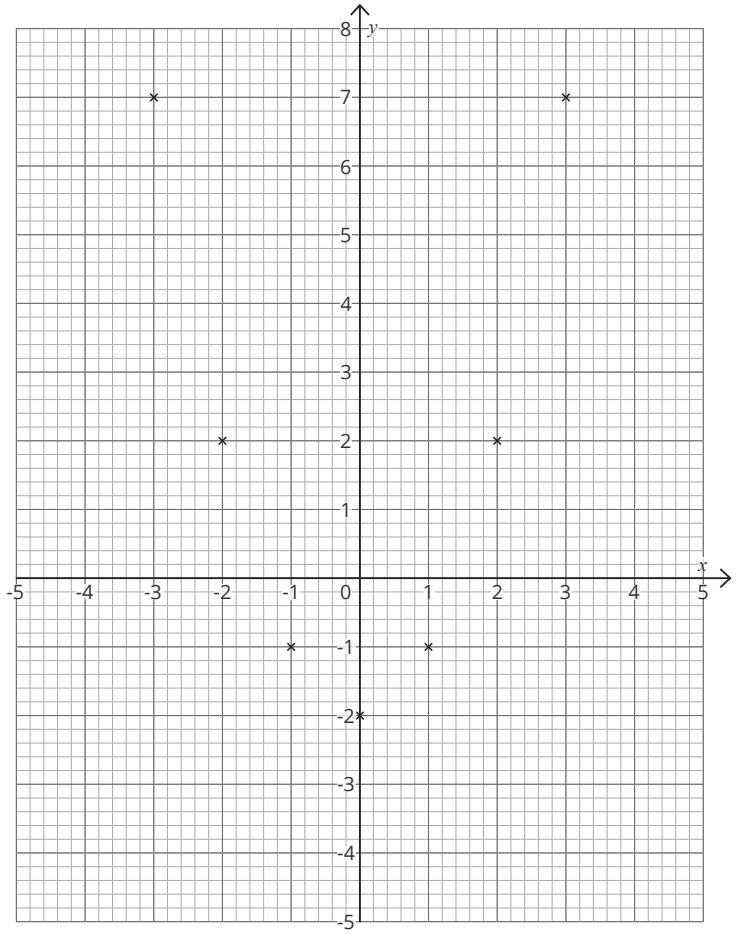
$y = 7$

When $x = 2$:

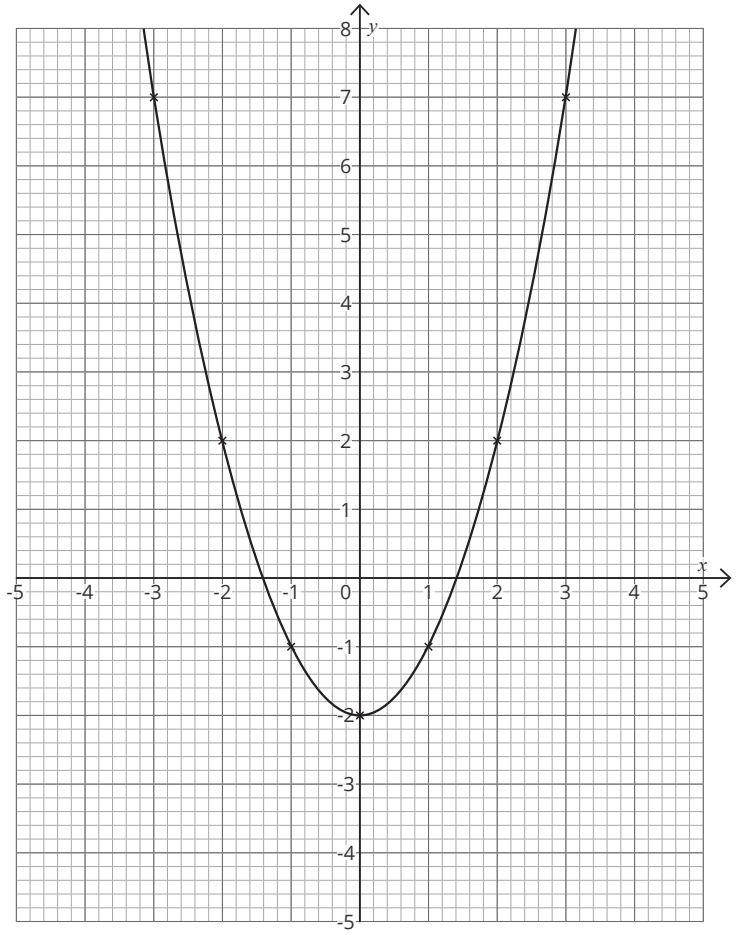
$y = 2^2 - 2$

$y = 2$

x	**-3**	-2	-1	0	1	**2**	3
y	**7**	2	-1	-2	-1	**2**	7

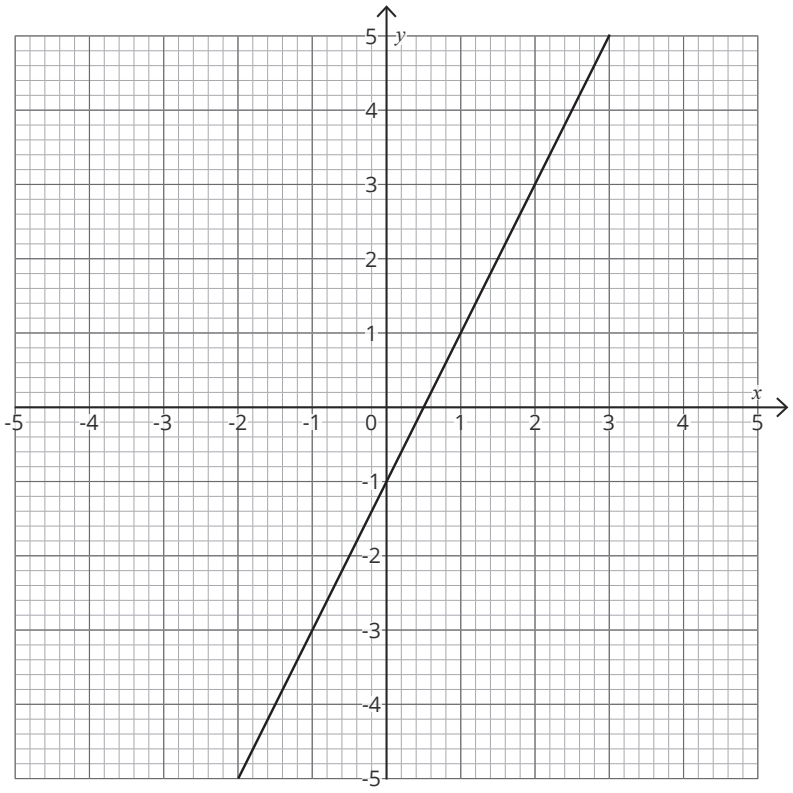


Step 3: Read the coordinate pairs off the table and plot them. In this example, they are (-3, 7), (-2, 2), (-1, -1) , (0, -2), (1, -1), (2, 2), (3, 7).



Step 4. Join the points together. With a linear graph, you use a ruler to draw a straight line extending to both edges of the axis. In this case, we are drawing a quadratic, so draw a smooth curve which joins all the points and extends to each edge of your axis. Do not join your points with straight lines.

Example 3: Find the y-intercept and gradient of the line below.



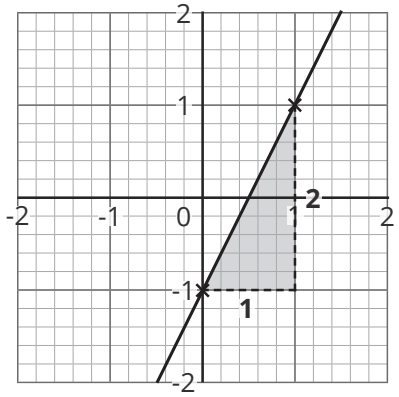
The y-intercept is the y-coordinate where the line crosses the y-axis. In this case, that is -1. Therefore, the y-intercept of this graph (sometimes known as c) is -1.

The gradient is the steepness of the graph – the bigger (further from 0) the gradient, the steeper the graph.

To find the gradient, start by choosing a point on the line. It is much easier if this point is on the grid lines: (0,-1) rather than (-0.75, -2.5).

Next, pick another point on the line, to the right of your first point. Again, it is easier if the second point is on the grid lines – the closer to your first point the better. We will pick (1, 1).

Look at the horizontal and vertical distances between these points:

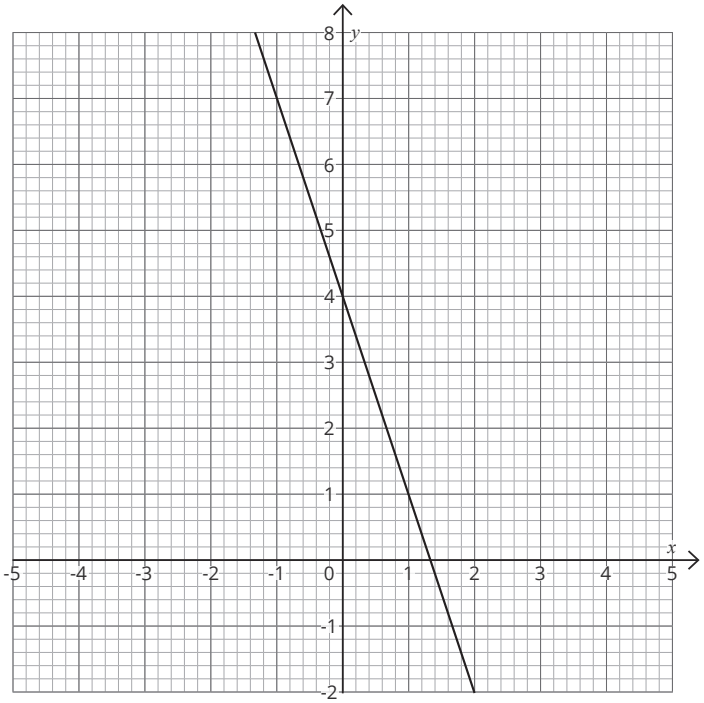


The gradient is the change in y-values (the vertical distance between your points), sometimes called 'rise', divided by the change in x-values (the horizontal distance between your points), sometimes called 'run':

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{2}{1} = 2$$

Finally, check if the graph is increasing (going up) or decreasing (going down). Looking from the left to the right, if the line is going up then the gradient is positive. If the line is going down, the gradient is negative. In this case, the graph is going up, so the gradient is positive. We can therefore say the gradient (sometimes called m) = 2.

Example 4: Find the equation of the line given below:



The equation of a linear graph is given in the form:

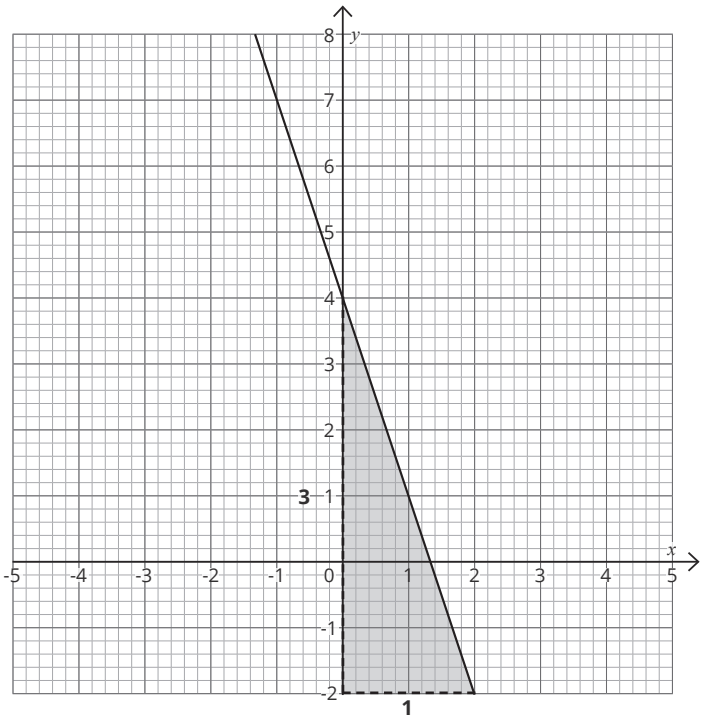
$$y = mx + c$$

Where m is the gradient and c is the y-intercept. Our line crosses the y-axis at 4:

$$c = 4.$$

If we find our gradient as before, we get a rise of 3 and a run of 1:

$$m = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$$



However, this time our line is decreasing, which means our gradient is negative:

$$m = -3$$

If we substitute these two values into the equation above, we get:

$$y = -3x + 4$$

Your Turn

1. Complete the table of values for the function $y = x - 3$

x	-2	-1	0	1	2	3	4
y	-5						1

2. Complete the table of values for the function $y = 3x + 2$

x	-3	-2	-1	0	1	2	3
y	-7	-4					

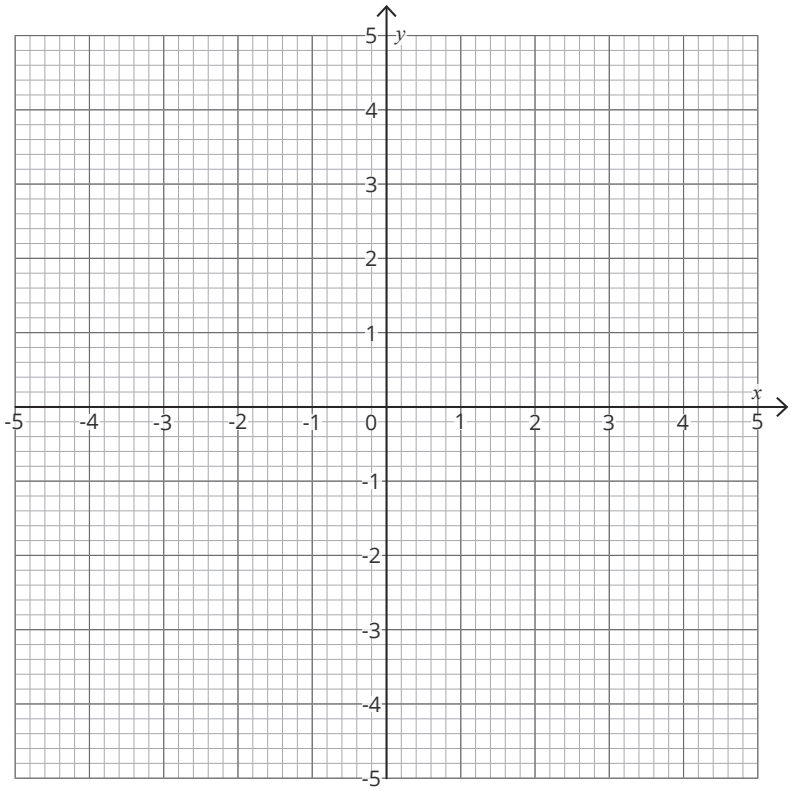
3. Complete the table of values for the function $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8					

4. Complete the table of values for the function $y = 3x^2 - 2$

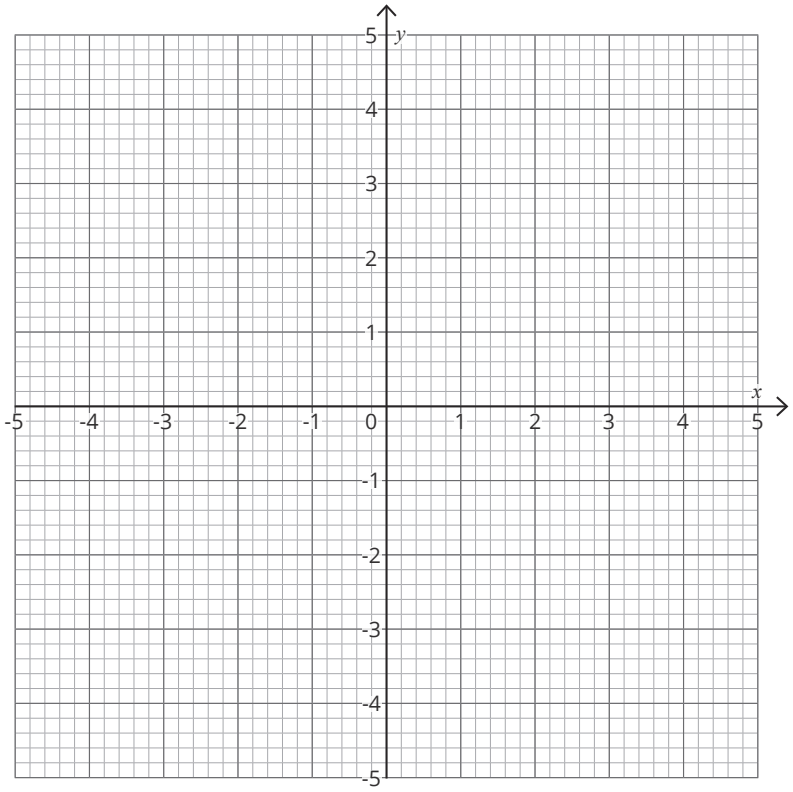
x	-3	-2	-1	0	1	2	3
y				-2		10	

5. On the grid below, draw the equation of the line $y = -x$.



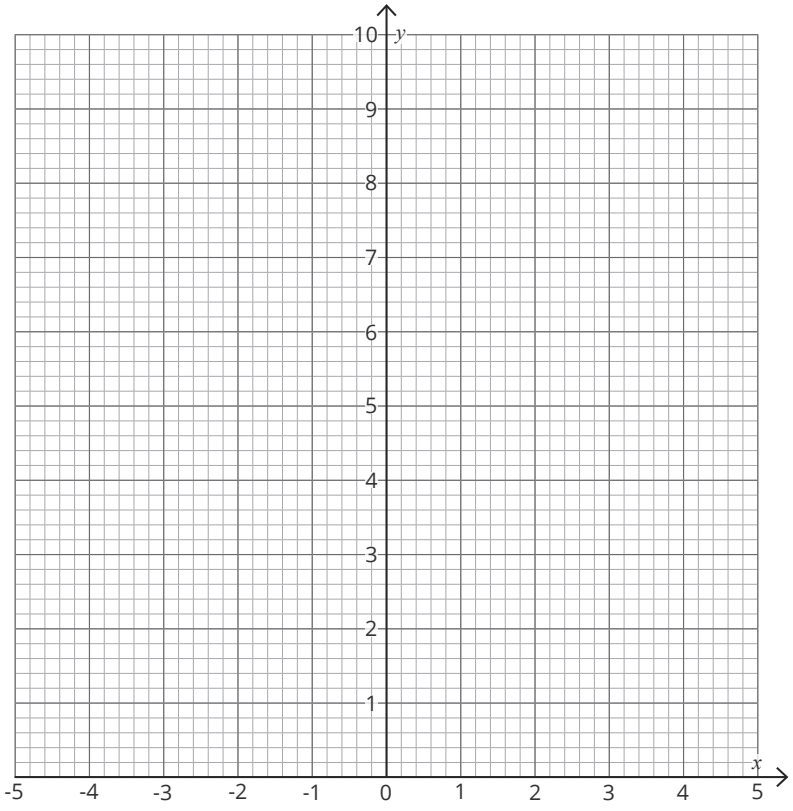
6. Complete the table of values for the function $y = 2x + 2$, then plot the graph on the axes below.

x	-3	-2	-1	0	1	2	3
y							

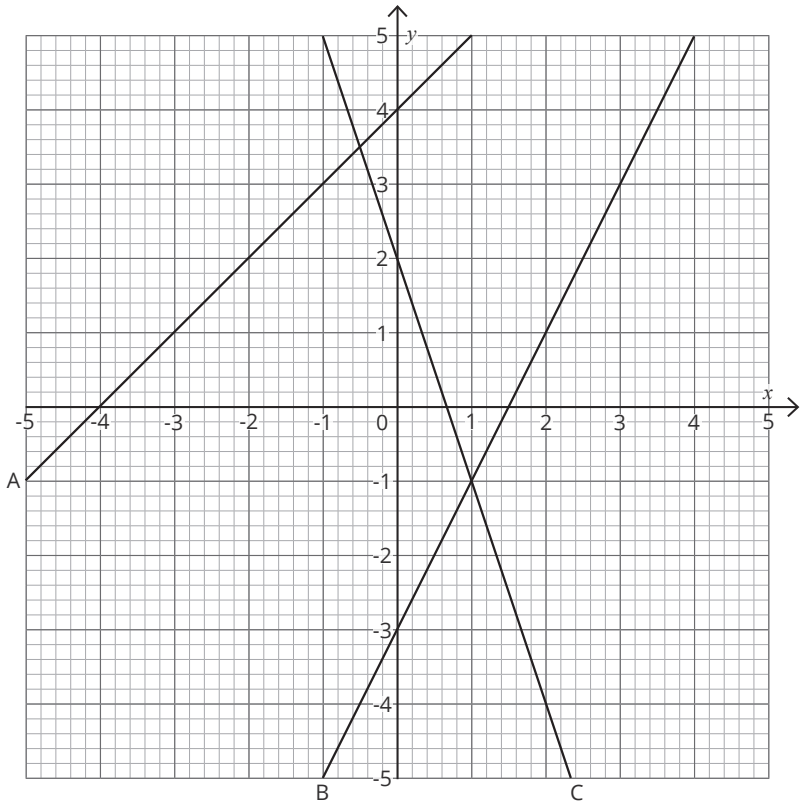


7. Complete the table of values for the function $y = x^2 + 1$, then plot the graph on the axes below.

x	-3	-2	-1	0	1	2	3
y							

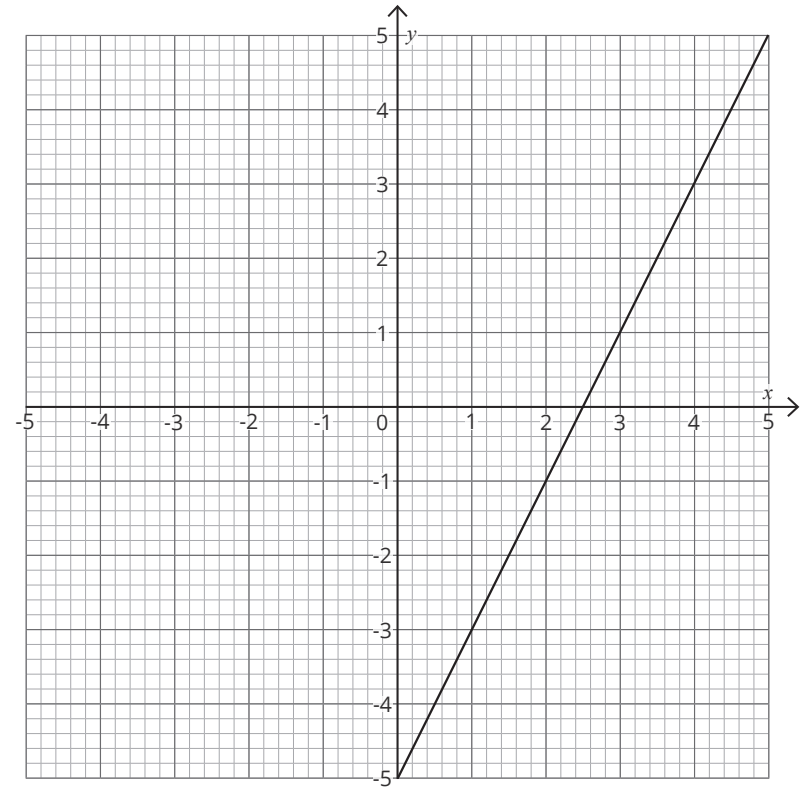


8. Identify the gradient (m) of each line below.



A. _____ B. _____ C. _____

9. Identify the equation of the line.



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Challenge

Plot a line with a gradient of 2 which crosses the point (2, 8). What is the equation of this line?